

NAG Toolbox for MATLAB

f07ar

1 Purpose

f07ar computes the LU factorization of a complex m by n matrix.

2 Syntax

```
[a, ipiv, info] = f07ar(a, 'm', m, 'n', n)
```

3 Description

f07ar forms the LU factorization of a complex m by n matrix A as $A = PLU$, where P is a permutation matrix, L is lower triangular with unit diagonal elements (lower trapezoidal if $m > n$) and U is upper triangular (upper trapezoidal if $m < n$). Usually A is square ($m = n$), and both L and U are triangular. The function uses partial pivoting, with row interchanges.

4 References

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

- 1: **a(lda,*)** – complex array

The first dimension of the array **a** must be at least $\max(1, \mathbf{m})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The by n coefficient matrix A .

5.2 Optional Input Parameters

- 1: **m** – int32 scalar

m , the number of rows of the matrix A .

Constraint: $\mathbf{m} \geq 0$.

- 2: **n** – int32 scalar

Default: The second dimension of the array **a**.

n , the number of columns of the matrix A .

Constraint: $\mathbf{n} \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

lda

5.4 Output Parameters

- 1: **a(lda,*)** – complex array

The first dimension of the array **a** must be at least $\max(1, \mathbf{m})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The factors L and U from the factorization $A = PLU$; the unit diagonal elements of L are not stored.

2: **ipiv**(*) – **int32** array

Note: the dimension of the array **ipiv** must be at least $\max(1, \min(\mathbf{m}, \mathbf{n}))$.

The pivot indices. Row i of the matrix A was interchanged with row **ipiv**(i), for $i = 1, 2, \dots, \min(m, n)$.

3: **info** – **int32** scalar

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info = $-i$

If **info** = $-i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **m**, 2: **n**, 3: **a**, 4: **lda**, 5: **ipiv**, 6: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

info > 0

If **info** = i , $U(i, i)$ is exactly zero. The factorization has been completed, but the factor U is exactly singular, and division by zero will occur if it is used to solve a system of equations.

7 Accuracy

The computed factors L and U are the exact factors of a perturbed matrix $A + E$, where

$$|E| \leq c(\min(m, n))\epsilon P|L||U|,$$

$c(n)$ is a modest linear function of n , and ϵ is the *machine precision*.

8 Further Comments

The total number of real floating-point operations is approximately $\frac{8}{3}n^3$ if $m = n$ (the usual case), $\frac{4}{3}n^2(3m - n)$ if $m > n$ and $\frac{4}{3}m^2(3n - m)$ if $m < n$.

A call to this function with $m = n$ may be followed by calls to the functions:

f07as to solve $AX = B$, $A^T X = B$ or $A^H X = B$;

f07au to estimate the condition number of A ;

f07aw to compute the inverse of A .

The real analogue of this function is f07ad.

9 Example

```
a = [complex(-1.34, +2.55), complex(0.28, +3.17), complex(-6.39, -2.2),
      complex(0.72, -0.92);
      complex(-0.17, -1.41), complex(3.31, -0.15), complex(-0.15, +1.34),
      complex(1.29, +1.38);
```

```

        complex(-3.29, -2.39), complex(-1.91, +4.42), complex(-0.14, -1.35),
        complex(1.72, +1.35);
        complex(2.41, +0.39), complex(-0.5600000000000001, +1.47), complex(-
0.83, ...
        -0.6899999999999999), complex(-1.96, +0.67)];
[aOut, ipiv, info] = f07ar(a)

```

```

aOut =
  -3.2900 - 2.3900i   -1.9100 + 4.4200i   -0.1400 - 1.3500i    1.7200 +
1.3500i
   0.2376 + 0.2560i    4.8952 - 0.7114i   -0.4623 + 1.6966i    1.2269 +
0.6190i
  -0.1020 - 0.7010i   -0.6691 + 0.3689i   -5.1414 - 1.1300i    0.9983 +
0.3850i
  -0.5359 + 0.2707i   -0.2040 + 0.8601i    0.0082 + 0.1211i    0.1482 -
0.1252i
ipiv =
      3
      2
      3
      4
info =
      0

```
